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A TIME DELAY CONTROLLER for MAGNETIC BEARINGS

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Abstract

The control of systems with unknown dynamics and unpredictable disturbances has raised some challenging problems. This is particularly important when high system peformance is to be guaranteed at all times. Recently, the Time Delay Control has been suggested as an alternative control scheme. The proposed control system does not require an explicit plant model nor does it depend on the estimation of specific plant parameters. Rather, it combines adaptation with past observations to directly estimate the effect of the plant dynamics.

This paper formulates a control law for a class of dynamic systems and then presents a sufficient condition for control system stability. The derivation is based on the bounded input-bounded output stability approach using L_∞ function norms. The control scheme is implemented on a five degrees-of-freedom high speed and high precision magnetic bearing. The control performance is evaluated using step responses, frequency responses and disturbance rejection properties. The experimental data show an excellent control performance despite the system complexity.

1 Introduction

Some classical control methods deal with well known linear time-invariant systems. In many applications, however, some relevant part of the system maybe unknown, time varying, or nonlinear. Controlled systems are thus often limited to operating in only a small portion of their available range. For example, servo motors must operate in the linear part of their range for accurate control. Restrictions such as these have led to the development of control techniques that deal with such complexities.

Several types of modern control strategies have been developed to deal with nonlinear, time-variant systems. One of the first methods to accommodate nonlinear systems was Model Reference Control. This technique employs a model of the system and uses the difference between the model response and the plant response as the input signal to the plant [18]. The model is either a physical model or a simulated system on a computer. Although it has no variable parameters, it is very useful for either specifying desired performance or for the observation of unaccessable states. A drawback in this technique is that it requires knowledge of the full dynamic model and system limitations. When perfect cancellation of the system nonlinearities is not achieved due to imperfect modeling or inaccurate parameter values, the dynamic performance of the plant may be degraded to the point of closed loop instability [22].

Another advanced technique is Adaptive Control. An adaptive system measures a certain index of performance which is a function of the inputs, states and/or outputs of the system. From the comparison of the measured index of performance with a set of given ones, the adaptation mechanism modifies the parameters of the controller or the set of given ones [4,14,16,21]. There are several classes of adaptive control. A very common variation uses a desired reference model as a basis for comparison and is termed Model Reference Adaptive Control (MRAC). In the direct MRAC, no attempt is made to identify the plant parameters. Controller parameters are directly updated. In Self-Tuning control, plant model parameters are identified/modified and the controller action is automatically updated according to a

fixed regulator design. Another approach generates the control action in part by an adaptive feedforward controller which "behaves" as the "inverse" of the plant [22]. All adaptive controllers share the distinguishing feature of system identification followed by variation of parameters to maintain desired performance. A drawback of adaptation is that it is generally slow and computationally intense. Often the environment changes faster than the system, causing performance degradation or even instability. Other references on adaptive control include [8,9,11,12,19,20,24].

Other control methods, such as Variable Structure Controllers, take totally different strategies to achieve stability in nonlinear systems. This type of controller utilizes state feedback in a control law which switches the structure of the closed loop system between trajectories which may themselves be unstable or marginally stable but when combined by the control law in a switching technique, result in a system which is stable. A method of switching called "sliding mode", described in [23,26,27,37], arranges the switching so that ideally the system remains on one of the switching lines (or surfaces) as it "slides" stably toward the origin of the phase plane. Real systems, however, take time to switch trajectories, resulting in periods of infinite frequency, or no control, as the system switches from one trajectory to another while attempting to remain on the switching line. This high frequency chattering undesirably excites high frequency dynamics.

Systems which are capable of recognizing the familiar features and patterns of a situation and which use past experiences in behaving in an optimal fashion are called Learning Systems. A learning system, when presented with a novel situation, learns how to behave by an adaptive approach. Then if the system experiences the same situation, it will recognize and behave optimally without going through the same adaptive approach. An advantage is that the system need not be identified in every environmental situation, making the response time faster under situations that have already been learned. A drawback is that such systems often require repetitive trial and error to bring them into an operating state [1,25]. A large list of references on methods of control mentioned above can be found in [13] and [17].

Another method, Time Delay Control (TDC) proposed in references [30,31,32,33], depends neither on estimation of specific parameters, repetitive actions, infinite switching frequencies, or discontinuous control. It employs, rather, direct estimation of the effect of the plant dynamics through the use of time delay. The controller uses the gathered information to cancel the unknown dynamics and disturbances simultaneously and then inserts the desired dynamics into the plant. The TDC employs past observation of the system response and control inputs to directly modify the control actions rather than adjust the controller gains. It updates its observation of the system every sampling period, therefore, estimation of the plant dynamics is dependent upon the sampling frequency. The TDC has a similar feature as the learning control algorithm proposed in reference [10]. This learning control algorithm is applicable for nonlinear systems with linear input action. It updates the control action in each learning trial by comparing the state derivative of the actual trajectory with that of the desired reference trajectory in the previous trial. Time Delay Control differs from this approach in that the control action is updated at each instant based on recent past. This paper uses the concepts developed in references [30,31,32,33] to explore the potentials and limitations of the TDC approach.

The TDC control algorithm leads to systems that have a similar form to that of time delay systems. These systems, which are also referred to as time-lag or retarded systems, are systems in which time delay exists between the cause and effect. In time delay systems, these delays arise as a result of delays existing in the hardware components or computation [5]. In our case, the time delay is a feature of the control algorithm. The mathematical formulation for such time delay systems leads to delayed differential equations. A special class of these equations are referred to as integral-differential equations which were studied by Volterra [29]. Volterra was the first to study such systems and developed the theory to investigate the consequences of time delay. Several other researchers have contributed to the development of the general theory of the Volterra type. Reference [15] provides several references of contributors to delayed differential equations including historical perspective of control theory and developments of time delay.

The Time Delay Control was originally formulated in [30] for a class of nonlinear systems with linear input action. The control algorithm has been applied to robot manipulators and servo systems with very satisfactory results even under large system parameter variations and disturbances [30,31,32,33,35]. Stability and convergence analysis was also performed for linear SISO systems [34].

This paper formulates a control law for a class of dynamic systems with nonlinear input action and then presents a sufficient condition for control system stability. The derivation is based on the bounded input-bounded output stability approach using L_∞ function norms. The control scheme is implemented on a five degrees-of-freedom high speed and high precision magnetic bearing. The control performance is evaluated using step responses, frequency responses and disturbance rejection properties. The experimental data show an excellent control performance despite the system complexity.

2 Time Delay Control

In this paper we are concerned with a class of systems described by the following differential equations,

$$\dot{x}(t) = F(x,t) + G(x,u,t) + D(t) \tag{1}$$

where $x(t) \in \mathcal{R}^n$ and $u(t) \in \mathcal{R}^r$ are the system state vector and control input vector respectively. $F(x,t)$, $G(x,u,t)$ and $D(t)$ are vector functions with appropriate dimensions and represent respectively known dynamics, unknown dynamics and disturbances. The variable t represents time. In order to transform the system into a familiar form, Equ.(1) can be written as

$$\dot{x}(t) = F(x,t) + H(x,u,t) + Bu(t) \tag{2}$$

where the new term $H(x,u,t)$ is defined as

$$H(x,u,t) = G(x,u,t) + D(t) - Bu(t) \tag{3}$$

and B is a matrix to be selected by the designer. A reference model that generates the desired trajectory is chosen as a linear time invariant system,

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{4}$$

where $x_m(t) \in \mathcal{R}^n$ is a reference model state vector, $r(t) \in \mathcal{R}^r$ is a reference input. A_m and B_m are constant matrices with appropriate dimensions.

The class of systems considered in this paper includes systems that satisfy a matching condition. It was shown in references [30,32,33] that systems in a special canonical form satisfy the matching condition. These systems can be partitioned as follows

$$x = \begin{bmatrix} x_q \\ \dots \\ x_r \end{bmatrix} \quad ; \quad F(x,t) = \begin{bmatrix} x_s \\ \dots \\ F_r(x,t) \end{bmatrix}$$

$$H(x,u,t) = \begin{bmatrix} 0 \\ \dots \\ H_r(x,u,t) \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 \\ \dots \\ B_r \end{bmatrix}$$

where the partial states are $x_q \in \mathcal{R}^{n-r}, x_r \in \mathcal{R}^r, x_s = [x_{r+1}, x_{r+2} \dots, x_n]' \in \mathcal{R}^{n-r}$. The vector functions have the following dimensions $F_r(x,t)$, $H_r(x,u,t) \in \mathcal{R}^r, B \in \mathcal{R}^{n \times r}$ and $B_r \in \mathcal{R}^{r \times r}$ is of rank r. The matrices involved in the reference model of Equ. (4) are also partitioned in the same manner,

$$A_m = \begin{bmatrix} 0 & | & I_q \\ & \dots & \\ & A_{mr} & \end{bmatrix} , B_m = \begin{bmatrix} 0 \\ \dots \\ B_{mr} \end{bmatrix}$$

where $I_q \in \mathcal{R}^{(n-r) \times (n-r)}$, $A_{mr} \in \mathcal{R}^{r \times n}$, $B_{mr} \in \mathcal{R}^{r \times r}$, $B_m \in \mathcal{R}^{n \times r}$ and $r(t) \in \mathcal{R}^r$. Furthermore assume a feedback matrix K of the form,

$$K = \begin{bmatrix} 0 \\ \dots \\ K_r \end{bmatrix}$$

where $K \in \mathcal{R}^{n \times n}$ and $K_r \in \mathcal{R}^{r \times n}$. The objective is to generate a control action u that forces the error to vanish according to

$$\dot{e} = (A_m + K)e = A_e e \tag{5}$$

The control action that combines past observations with adaptation for systems described by Equ. (2) is given by

$$u(t) = B^+[-\dot{x}(t-L) + F(x,t-L) - F(x,t)$$

$$+A_m x(t) + B_m r(t) + Bu(t-L) - Ke(t)] \tag{6}$$

where the parameter L represents the time delay [30,32,33], the error vector e is defined as the difference between the plant and the reference model state vectors,

$$\mathbf{e} = \mathbf{x}_m - \mathbf{x} \tag{7}$$

The term $\mathbf{B}^+$ is the pseudo-inverse matrix defined as $\mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T$. For the special canonical form considered, $\mathbf{B}^+$ is given by

$$\mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T = \left\{ \begin{bmatrix} \mathbf{0} \\ \cdots \\ \mathbf{B}_r \end{bmatrix}^T \begin{bmatrix} \mathbf{0} \\ \cdots \\ \mathbf{B}_r \end{bmatrix} \right\}^{-1} [\mathbf{0} \vdots \mathbf{B}_r^T]$$

$$= (\mathbf{B}_r^T\mathbf{B}_r)^{-1}[\mathbf{0} \vdots \mathbf{B}_r^T] = \mathbf{B}_r^{-1}[\mathbf{0} \vdots \mathbf{I}_r] = [\mathbf{0} \vdots \mathbf{B}_r^{-1}]$$

The control action now reduces to

$$\begin{aligned} \mathbf{u}(t) = {} & \mathbf{B}_r^{-1}[-\dot{\mathbf{x}}_r(t-L) + \mathbf{F}_r(\mathbf{x}, t-L) - \mathbf{F}_r(\mathbf{x}, t) + \mathbf{A}_{mr}\mathbf{x}(t) \\ & + \mathbf{B}_r\mathbf{u}(t-L) + \mathbf{B}_{mr}\mathbf{r}(t) - \mathbf{K}_r\mathbf{e}(t)] \end{aligned} \tag{8}$$

Note that this control law is a special case of a general algorithm which uses convolutions for estimating unknown system dynamics [36].

The objective of this research is to be able to control such systems and guarantee performance despite the presence of large dynamic variations in $\mathbf{G}(\mathbf{x}, \mathbf{u}, t)$ and large unexpected disturbances in $\mathbf{D}(t)$.

As described in [31], each term in Equ.(8) has the following meaning: (1) $\mathbf{B}_r^{-1}$, cancels the control matrix $\mathbf{B}_r$, (2) the term $-\mathbf{F}(\mathbf{x}, t) - \dot{\mathbf{x}}(t-L) + \mathbf{F}(\mathbf{x}, t-L) + \mathbf{B}\mathbf{u}(t-L)$ attempts to cancel the undesired known nonlinear dynamics $\mathbf{F}(\mathbf{x}, t)$, the unknown nonlinear dynamics and the unexpected disturbances $\mathbf{H}(\mathbf{x}, t)$, (3) the term $\mathbf{A}_m\mathbf{x} + \mathbf{B}_m\mathbf{r}$ inserts the desired dynamics of the reference model, and (4) the error feedback term $-\mathbf{K}\mathbf{e}$ adjusts the error dynamics. Thus this controller observes the current state, the state derivatives (estimates) and the inputs of the system at time $t - L$, one step into the past, and determines the best control action that should be commanded at time t. The scheme used in the time delay control is reminiscent of numerical methods used to solve differential equations.

3 Stability Analysis

3.1 Error dynamics

As indicated in [32,33], the stability of such control systems using time delay depends on the delay parameter L, the control gains $\mathbf{K}$, the speed of the response of the plant and the speed of response of the reference trajectory. The method used to perform this analysis is based on the bounded input bounded output stability procedure. In what follows, we discuss the stability analysis for two situations pertaining to whether the control distribution matrix $\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$ is constant and known or unknown. In order to perform the stability analysis, we formulate the governing equations for the error dynamics. First using the control action of Equ.(8), the plant equations of Equ.(1) become

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} \mathbf{x}_s \\ \cdots \\ \mathbf{F}_r(\mathbf{x}, t) + \mathbf{H}_r(\mathbf{x}, \mathbf{u}, t) + [-\dot{\mathbf{x}}_r(t-L) + \mathbf{F}_r(\mathbf{x}, t-L) - \mathbf{F}_r(\mathbf{x}, t) \\ + \mathbf{B}_r\mathbf{u}(t-L) + \mathbf{A}_{mr}\mathbf{x}(t) + \mathbf{B}_{mr}\mathbf{r}(t) - \mathbf{K}_r\mathbf{e}(t)] \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{x}_s \\ \cdots \\ \mathbf{H}_r(\mathbf{x}, \mathbf{u}, t) - \mathbf{H}_r(\mathbf{x}, \mathbf{u}, t-L) + \mathbf{A}_{mr}\mathbf{x}(t) + \mathbf{B}_{mr}\mathbf{r}(t) - \mathbf{K}_r\mathbf{e}(t) \end{bmatrix} \end{aligned}$$

The previously defined error e of Equ.(7) is now governed by

$$\dot{\mathbf{e}}(t) = (\mathbf{A}_m + \mathbf{K})\mathbf{e}(t) + \mathbf{H}(\mathbf{x}, \mathbf{u}, t-L) - \mathbf{H}(\mathbf{x}, \mathbf{u}, t) \tag{9}$$

where the second and third terms are forcing functions due to the unknown system dynamics and unpredictable disturbances.

Rewriting Equ. (9) as

$$\dot{e}(t) = (\mathbf{A}_m + \mathbf{K})e(t) + \mathbf{p}(t) \tag{10}$$

where

$$\mathbf{p}(t) = \left[\begin{array}{c} 0 \\ --- \\ \mathbf{p}_r(t) \end{array} \right]$$

and

$$\mathbf{p}_r(t) = \mathbf{H}_r(\mathbf{x}, \mathbf{u}, t - L) - \mathbf{H}_r(\mathbf{x}, \mathbf{u}, t)$$

One may ask the question: what conditions does the vector $\mathbf{p}(t)$ have to satisfy for the system to be stable? A sufficient condition for stability will be derived in the next section.

3.2 Sufficient conditions for stability

This section presents a general solution to this multi-input multi-outp t control problem We will use the bounded input-bounded output approach based on L_∞ norms in order to deri e sufficient conditions for stability. We now consider the governing differential of the error as given by Equ.(10) an I its correponding time response,

$$e(t) = e^{(\mathbf{A}_m + \mathbf{K})t}e(0) + \int_0^t e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)}\mathbf{p}(\tau)d\tau$$

We will use $\|(.)_T\|$ to indicate the norm of the time trancated function (.) and $\|(.)\|_i$ for the induced matrix norm. Taking the norm of the error [28],

$$\|e_T\| \leq \| \left(e^{(\mathbf{A}_m + \mathbf{K})t} \right)_T \|_i \|e(0)\| + \sup_{t \in [0,T]} \int_0^t \| e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)} \|_i \|\mathbf{p}(\tau)\| d\tau$$

$$\leq \| \left(e^{(\mathbf{A}_m + \mathbf{K})t} \right)_T \|_i \|e(0)\| + \|\mathbf{p}_T\| \sup_{t \in [0,T]} \int_0^t \| e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)} \|_i d\tau$$

The desired error dynamics given by $(\mathbf{A}_m + \mathbf{K})$ are always chosen to be asymptotically stable. This implies that there exist finite positive constants m, λ such that

$$\| e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)} \|_i \leq me^{-\lambda(t-\tau)} \qquad \forall \tau, t > \tau$$

which implies

$$\sup_{t \in [0,T]} \int_0^t \| e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)} \|_i d\tau \leq \sup_{t \in [0,T]} \frac{m}{\lambda}(1 - e^{-\lambda t}) = \frac{m}{\lambda}$$

$$\| (e^{(\mathbf{A}_m + \mathbf{K})t})_T \|_i = \sup_{t \in [0,T]} \|(e^{(\mathbf{A}_m + \mathbf{K})t})\|_i$$

$$\leq \sup_{t \in [0,T]} me^{-\lambda t} = m$$

Therefore, the norm of the error is bounded

$$\|e_T\| \leq \alpha + \beta\|\mathbf{p}_T\| \tag{11}$$

where

$$\alpha = m\| e(0)\|, \quad \beta = \frac{m}{\lambda}$$

In order to be more specific on these stability conditions, we need to expand the forcing term $\mathbf{p}(t)$. We can rewrite $\mathbf{p}_r(t)$ as

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$$\mathbf{p}_r(t) = \mathbf{H}_r(\mathbf{x}(t-L),\mathbf{u}(t-L),t-L) - \mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t)$$

$$+\mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t) - \mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t),t)$$

$$\|[\mathbf{p}(t)]_T\| = \|[\mathbf{p}_r(t)]_T\| \le \|[\mathbf{H}_r(\mathbf{x}(t-L),\mathbf{u}(t-L),t-L) \tag{12}$$

$$-\mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t)]_T\|$$

$$+\|[\mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t) - \mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t),t)_T\|$$

Assuming that the function $\mathbf{H}_r(\mathbf{x},\mathbf{u},t)$ is continuous and differentiable, the Mean Value Theorem yields [6]

$$\|[\mathbf{H}_r(\mathbf{x}(t-L),\mathbf{u}(t-L),t-L) - \mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t)]_T\|$$

$$\le \left\| \left[\left(\frac{\partial \mathbf{H}_r}{\partial t} + \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{x}} \right)(\mathbf{x}(\zeta),\mathbf{u}(t-L),\zeta) \right]_T \right\| L \tag{13}$$

$$\|[\mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t-L),t) - \mathbf{H}_r(\mathbf{x}(t),\mathbf{u}(t),t)]_T\|$$

$$\le \left\| \left[\frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}(\mathbf{x}(t),\delta,t)(\mathbf{u}(t) - \mathbf{u}(t-L)) \right]_T \right\| \tag{14}$$

where $\zeta \in (t-L,t)$ and $\delta \in (\mathbf{u}(t-L),\mathbf{u}(t))$.

Equations (13) and (14) involve terms in $\mathbf{x}$ and $\mathbf{u}$. To express them in terms of $\mathbf{e}$, the following expressions are used,

$$\frac{\partial \mathbf{H}_r}{\partial t} + \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{x}} = \frac{\partial \mathbf{H}_r}{\partial t} + \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{x}}_m - \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{e}} \tag{15}$$

$$\mathbf{u}(t) - \mathbf{u}(t-L) = \mathbf{B}_r^{-1}[-\dot{\mathbf{x}}_r(t-L) + \mathbf{F}_r(\mathbf{x},t-L) - \mathbf{F}_r(\mathbf{x},t)$$

$$+\mathbf{A}_{mr}\mathbf{x}(t) + \mathbf{B}_{mr}\mathbf{r}(t) - \mathbf{K}_r\mathbf{e}(t)]$$

$$= \mathbf{B}_r^{-1}[-\dot{\mathbf{x}}_{mr}(t-L) + \dot{\mathbf{e}}_r(t-L) + \mathbf{F}_r(\mathbf{x},t-L) - \mathbf{F}_r(\mathbf{x},t) \tag{16}$$

$$+\dot{\mathbf{x}}_{mr}(t) - (\mathbf{A}_{mr} + \mathbf{K}_r)\mathbf{e}(t)]$$

since $\dot{\mathbf{e}}_r = \dot{\mathbf{x}}_{mr} - \dot{\mathbf{x}}_r$ and $\dot{\mathbf{x}}_{mr}(t) = \mathbf{A}_{mr}\mathbf{x}_m(t) + \mathbf{B}_{mr}\mathbf{r}(t)$. Substitution of (13), (14), (15) and (16) in (2) and some algebraic manipulation yields,

$$\|\mathbf{p}_T\| \le \left\| \frac{\partial \mathbf{H}_r}{\partial t} + \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{x}}_m \right\| L + \left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}} \right\|_i L \|\dot{\mathbf{e}}_T\|$$

$$+\left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1}(\mathbf{A}_{mr} + \mathbf{K}_r) \right\|_i \|\mathbf{e}_T\|$$

$$+\left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1} \right\|_i \|(\dot{\mathbf{x}}_{mr}(t) - \dot{\mathbf{x}}_{mr}(t-L))\| \tag{17}$$

$$+\left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1} \right\|_i \|\dot{\mathbf{e}}_T\|$$

$$+\left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1} \right\|_i \|[\mathbf{F}_r(\mathbf{x},t-L) - \mathbf{F}_r(\mathbf{x},t)]_T\|$$

$$\|[\mathbf{F}_r(\mathbf{x},t-L) - \mathbf{F}_r(\mathbf{x},t))]_T\| \le \left\| \left[\left(\frac{\partial \mathbf{F}_r}{\partial t} + \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\dot{\mathbf{x}} \right)(\mathbf{x}(\xi),\xi) \right]_T \right\| L \tag{18}$$

$$\le \left\| \frac{\partial \mathbf{F}_r}{\partial t} + \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\dot{\mathbf{x}}_m \right\| L + \left\| \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}} \right\|_i L \|\dot{\mathbf{e}}_T\|$$

where $\xi \in (t-L,t)$. Substituting Equ. (18) in Equ. (17) yields

$$\|\mathbf{p}_T\| \;\leq\; c_1 + c_2\,\|\mathbf{e}_T\| + c_3\,\|\dot{\mathbf{e}}_T\| \tag{19}$$

where the constants c_1, c_2 and c_3 can be identified as

$$c_1 \;=\; \left\|\frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1}\right\|_i \left[L \left\|\frac{\partial \mathbf{F}_r}{\partial t} + \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\dot{\mathbf{x}}_m\right\| + \|\dot{\mathbf{x}}_{mr}(t) - \dot{\mathbf{x}}_{mr}(t - L)\| \right]$$
$$+\; L \left\|\frac{\partial \mathbf{H}_r}{\partial t} + \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\dot{\mathbf{x}}_m\right\|$$

$$c_2 \;=\; \left\|\frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1}(\mathbf{A}_{mr} + \mathbf{K}_r)\right\|_i$$

$$c_3 \;=\; \left\|\frac{\partial \mathbf{H}_r}{\partial \mathbf{x}}\right\|_i L + \left\|\frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1}\right\|_i + \left\|\frac{\partial \mathbf{H}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1}\right\|_i \left\|\frac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\right\|_i L$$

We will assume later that the terms in the right hand side of the three equations listed above are bounded and hence the parameters c_1, c_2 and c_3 will be bounded. The norm of the error in Equ.(11) can now be evaluated and is found to be,

$$\|\mathbf{e}_T\| \;\leq\; \alpha + \beta\,\|\mathbf{p}_T\| \;\leq\; \alpha + \beta c_1 + \beta c_2\|\mathbf{e}_T\| + \beta c_3\,\|\dot{\mathbf{e}}_T\|$$

or

$$\|\mathbf{e}_T\| \;\leq\; \frac{\alpha + \beta c_1 + \beta c_3\,\|\dot{\mathbf{e}}_T\|}{(1 - \beta c_2)} \quad \text{if } \beta c_2 < 1 \tag{20}$$

The above equation relates the norms of the error and the error derivative. To obtain absolute bounds on the error another equation of this form is needed. The norm of the derivative of the error $\dot{\mathbf{e}}$, can be found from Equs. (10) and (19),

$$\|\dot{\mathbf{e}}_T\| \;\leq\; \|\mathbf{A}_m + \mathbf{K}\|_i\,\|\mathbf{e}_T\| + \|\mathbf{p}_T\|$$
$$\leq\; [\,\|\mathbf{A}_m + \mathbf{K}\|_i + c_2]\,\|\mathbf{e}_T\| + c_1 + c_3\,\|\dot{\mathbf{e}}_T\| \tag{21}$$

This condition can be stated as,

$$\|\dot{\mathbf{e}}_T\| \;\leq\; \frac{c_1}{(1 - c_3)} + \frac{[\|\mathbf{A}_m + \mathbf{K}\|_i + c_2]}{(1 - c_3)}\,\|\mathbf{e}_T\| \quad \text{if } c_3 < 1 \tag{22}$$

and substituting Equ.(20) one obtains

$$\|\dot{\mathbf{e}}_T\| \;\leq\; \frac{[\,\|\mathbf{A}_m + \mathbf{K}\|_i + c_2]\,(\alpha + \beta c_1 + \beta c_3\|\dot{\mathbf{e}}_T\|)}{\quad + (1 - \beta c_2)c_1 \quad} {(1 - c_3)(1 - \beta c_2)}$$

or

$$\|\dot{\mathbf{e}}_T\| \;\leq\; \frac{\|\mathbf{A}_m + \mathbf{K}\|_i(\alpha + \beta c_1) + c_1 + c_2\alpha}{[1 - \beta c_2 - c_3 - \beta c_3\,\|\mathbf{A}_m + \mathbf{K}\|_i]}$$

if $c_3 + \beta(c_2 + c_3\,\|\mathbf{A}_m + \mathbf{K}\|_i) < 1$ is satisfied. Using the expressions for the constants c_1, c_2 and c_3, we have

$$\left(\left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{x}} \right\|_i L + \left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} \right\|_i + \left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} \right\|_i \left\| \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}} \right\|_i L \right)$$

$$\left[1 + \frac{m}{\lambda} \|\mathbf{A}_m + \mathbf{K}\|_i \right] \tag{23}$$

$$+ \left\| \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} (\mathbf{A}_{mr} + \mathbf{K}_r) \right\|_i \frac{m}{\lambda} < 1$$

Based on the foregoing analysis and relations between the vector functions $\mathbf{H}_r$, $\mathbf{G}_r$, and $\mathbf{B}_r$ given by

$$\frac{\partial \mathbf{H}_r}{\partial \mathbf{x}} = \frac{\partial \mathbf{G}_r}{\partial \mathbf{x}} \quad \text{and} \quad \frac{\partial \mathbf{H}_r}{\partial \mathbf{u}} = \frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} - \mathbf{B}_r$$

the following sufficient condition follows,

Theorem 1 :

If the functions $\mathbf{F}(\mathbf{x}, t), \mathbf{G}(\mathbf{x}, \mathbf{u}, t), \mathbf{D}(t)$ are continuous and differentiable, and

If the Jacobian matrices $\frac{\partial \mathbf{F}_r}{\partial \mathbf{x}}, \frac{\partial \mathbf{G}_r}{\partial \mathbf{x}}, \frac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$, and the vector functions $\frac{\partial \mathbf{F}_r}{\partial t}, \frac{\partial \mathbf{G}_r}{\partial t}, \frac{\partial \mathbf{D}_r}{\partial t} \in L_\infty$, and

If the eigenvalues of the matrix $(\mathbf{A}_m + \mathbf{K})$ are in the left-half plane, and

If the following stability condition is satisfied

$$\left[1 + \frac{m}{\lambda} \|\mathbf{A}_m + \mathbf{K}\|_i \right] \left[\left\| \frac{\partial \mathbf{G}_r}{\partial \mathbf{x}} \right\|_i L \right.$$

$$+ \left\| \frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right\|_i \left(1 + \left\| \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}} \right\|_i L \right) \right] \tag{24}$$

$$+ \left\| \left(\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right)(\mathbf{A}_{mr} + \mathbf{K}_r) \right\|_i \frac{m}{\lambda} < 1$$

Then the time delay controller is stable, and the resultant bounds on the norms of the error, $\mathbf{e}$, and its derivative $\dot{\mathbf{e}}$ are

$$\|\mathbf{e}\| \leq \frac{\alpha + \beta c_1 - \alpha c_3}{[1 - \beta c_2 - c_3 - \beta c_3 \|\mathbf{A}_m + \mathbf{K}\|_i]} \tag{25}$$

$$\|\dot{\mathbf{e}}\| \leq \frac{\|\mathbf{A}_m + \mathbf{K}\|_i (\alpha + \beta c_1) + c_1 + c_2 \alpha}{[1 - \beta c_2 - c_3 - \beta c_3 \|\mathbf{A}_m + \mathbf{K}\|_i]}$$

The constants c_1, c_2, c_3 α and β are,

$$c_1 = \left\| \left(\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right) \right\|_i \left[L \left\| \frac{\partial \mathbf{F}_r}{\partial t} + \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}} \dot{\mathbf{x}}_m \right\| + \|\dot{\mathbf{x}}_{mr}(t) - \mathbf{x}_{mr}(t - L)\| \right]$$

$$+ L \left\| \frac{\partial \mathbf{G}_r}{\partial t} + \frac{\partial \mathbf{G}_r}{\partial \mathbf{x}} \dot{\mathbf{x}}_m \right\|$$

$$c_2 = \left\| \left(\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right)(\mathbf{A}_{mr} + \mathbf{K}_r) \right\|_i$$

$$c_3 = \left\| \frac{\partial \mathbf{G}_r}{\partial \mathbf{x}} \right\|_i L + \left\| \frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right\|_i \left(1 + \left\| \frac{\partial \mathbf{F}_r}{\partial \mathbf{x}} \right\|_i L \right)$$
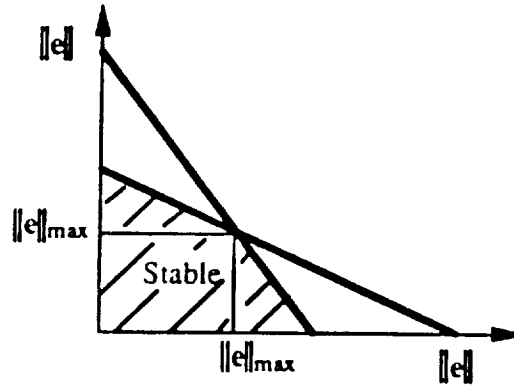
$$\alpha = m \|\mathbf{e}(0)\|$$

$$\beta = \frac{m}{\lambda}$$

Figure 1: Region of Stability in the $\|e_T\| - \|\dot{e}_T\|$ space

Condition (ii) implies that the rate of change of the functions $\mathbf{F}_r$, $\mathbf{G}_r$ and $\mathbf{D}_r$ with respect $t, \mathbf{x}$ and $\mathbf{u}$ are bounded. Condition (iii) implies that the desired error dynamics are chosen to be stable. Condition (iv) relates the time delay L, the rates of change of $\mathbf{F}_r$ and $\mathbf{G}_r$ with respect to $\mathbf{x}$ and $\mathbf{u}$, and the desired error dynamics specified by $(\mathbf{A}_m + \mathbf{K})$.

The bounds on the norms of the error, e and its derivative $\dot{e}$ are shown graphically in Figure 1. Equ. (20) yields a straight line as the boundary between stable and unstable regions. Similarly, Equ. (22) yields another straight line. The common region bounded by the two straight lines is the region of stability as shown. The absolute bounds $\|e\|_{max}$ and $\|\dot{e}\|_{max}$ shown in Figure 1 correspond to those given by Equ. (25).

3.3 Special cases and discussions

Condition (iv) stated in the proposition 1 can be rewritten in a more convenient and usable form. This simplified and convenient condition, however, is more conservative. Starting with condition (24) and after some algebraic manipulations we obtain,

$$\|\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{u}}\mathbf{B}_r^{-1} - \mathbf{I}\|_i < \frac{\left\{ 1 - \|\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{x}}\|_i L \left[1 + \tfrac{m}{\lambda} \|\mathbf{A}_m + \mathbf{K}\|_i \right] \right\}}{\left\{ \begin{array}{c} (1 + \|\tfrac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\|_i L) \\ + \tfrac{m}{\lambda} \left[\|\mathbf{A}_{mr} + \mathbf{K}_r\|_i \right. \\ \left. + \|\mathbf{A}_m + \mathbf{K}\|_i (1 + \|\tfrac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\|_i L) \right] \end{array} \right\}} \tag{26}$$

The condition (26) implies a bound on the variation of $\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$ relative to the controller gain matrix $\mathbf{B}_r$. The size of this bound is dependent on the delay time L, the norms of $\|\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{x}}\|_i$ and $\|\tfrac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\|_i$, and the desired characteristics of error dynamics given by $(\mathbf{A}_m + \mathbf{K})$. The smaller the delay time is and the smaller the bounds on $\|\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{x}}\|_i$ and $\|\tfrac{\partial \mathbf{F}_r}{\partial \mathbf{x}}\|_i$ are, then the larger the allowable size of bound is on the range of $\tfrac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$ relative to $\mathbf{B}_r$.

In the case of first order SISO systems the vector functions reduce to scalar functions $\mathbf{G}_r = g$, $\mathbf{B}_r = b$ and $\mathbf{F}_r = f$. Some interesting results are stated below for this class of systems. For first order SISO systems, we have

$$\|e^{(\mathbf{A}_m + \mathbf{K})(t-\tau)}\|_i = | e^{a_m + k)(t-\tau)} | = e^{(a_m + k)(t-\tau)}$$

$$m = 1$$

$$\lambda = -(a_m + k)$$

$$\|\mathbf{A}_{mr} + \mathbf{K}_r\|_i \ = \ \|\mathbf{A}_m + \mathbf{K}\|_i \ = \ |\, a_m + k \,| = \ -(a_m + k)$$

the stability condition (26) becomes

$$\left\| \frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} \mathbf{B}_r^{-1} - \mathbf{I} \right\|_i \ = \ \left\| \frac{\frac{\partial g}{\partial u}}{b} - 1 \right\|_i \ < \ \frac{\left(1 - 2\|\frac{\partial g}{\partial x}\| L \right)}{\left(3 + 2\|\frac{\partial f}{\partial x}\| L \right)} \tag{27}$$

The above condition implies

$$1 - \frac{\left(1 - 2\|\frac{\partial g}{\partial x}\| L \right)}{\left(3 + 2\|\frac{\partial f}{\partial x}\| L \right)} \ < \ \frac{\left(\frac{\partial g}{\partial u} \right)}{b} \ < \ 1 + \frac{\left(1 - 2\|\frac{\partial g}{\partial x}\| L \right)}{\left(3 + 2\|\frac{\partial f}{\partial x}\| L \right)} \tag{28}$$

with the lower bound on $\dfrac{\left(\frac{\partial g}{\partial u} \right)}{b}$ being always positive. For the case where $\dfrac{\partial g}{\partial u} = b$ the stability condition (28) becomes,

$$\|\frac{\partial g}{\partial x}\| L \ < \ \frac{1}{2} \tag{29}$$

This implies that as $\|\frac{\partial g}{\partial x}\|$ becomes large, the delay time L must be decreased to maintain stability which makes intuitive sense. From (28), it can be observed that for "sufficiently" small L $(L \to 0)$, the condition is reduced to the following limiting case,

$$\left\| \frac{\frac{\partial g}{\partial u}}{b} - 1 \right\|_i \ < \ \frac{1}{3} \tag{30}$$

$$\frac{2}{3} \ < \ \frac{\frac{\partial g}{\partial u}}{b} \ < \ \frac{4}{3}$$

The result indicates that stability is maintained for a variation of 66% of $\dfrac{\partial g}{\partial u}$ with respect to b.

When the control distribution matrix $\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$ is a constant and known, the controller gain matrix $\mathbf{B}_r$ may be chosen such that $\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}} = \mathbf{B}_r$. This enables exact cancellation of the known dynamics $\mathbf{F}_r$ and approximate cancellation of unknown dynamics and disturbances [33]. The stability condition (26) then reduces to a bound on delay time L in terms of $\frac{\partial \mathbf{G}_r}{\partial \mathbf{x}}$ and $(\mathbf{A}_m + \mathbf{K})$. The known dynamics $\mathbf{F}_r$, the control distribution matrix $\frac{\partial \mathbf{G}_r}{\partial \mathbf{u}}$ and the controller gain matrix $\mathbf{B}_r$ do not enter the stability condition because of exact cancellation.

4 Application: Control of a high speed and high precision magnetic bearing system

The magnetic system under consideration is a turbo molecular pump, a device used to create vaccum in special environments such as integrated circuit manufacturing. A schematic diagram of this pump is shown in Figure 2. The pump action is produced once the rotor, with blades attached to it, is spun by an induction motor. In order to minimize impurities and particle generation, the rotor is suspended magnetically in the X, Y and Z directions shown in the figure. Some information relevant to this design are summarized in Table 1. This system has five degrees which may be described by a differential equation of the form,

$$\frac{d}{dt} \begin{bmatrix} \mathbf{x}_q \\ \cdots \\ \mathbf{x}_r \end{bmatrix} = \begin{bmatrix} \mathbf{x}_r \\ \cdots\cdots \\ \mathbf{F}_r(\mathbf{x}, t) \end{bmatrix} + \begin{bmatrix} 0 \\ \cdots\cdots\cdots \\ \mathbf{G}_r(\mathbf{x}, \mathbf{u}, t) \end{bmatrix} + \begin{bmatrix} 0 \\ \cdots \\ \mathbf{D}_r(t) \end{bmatrix} \tag{31}$$

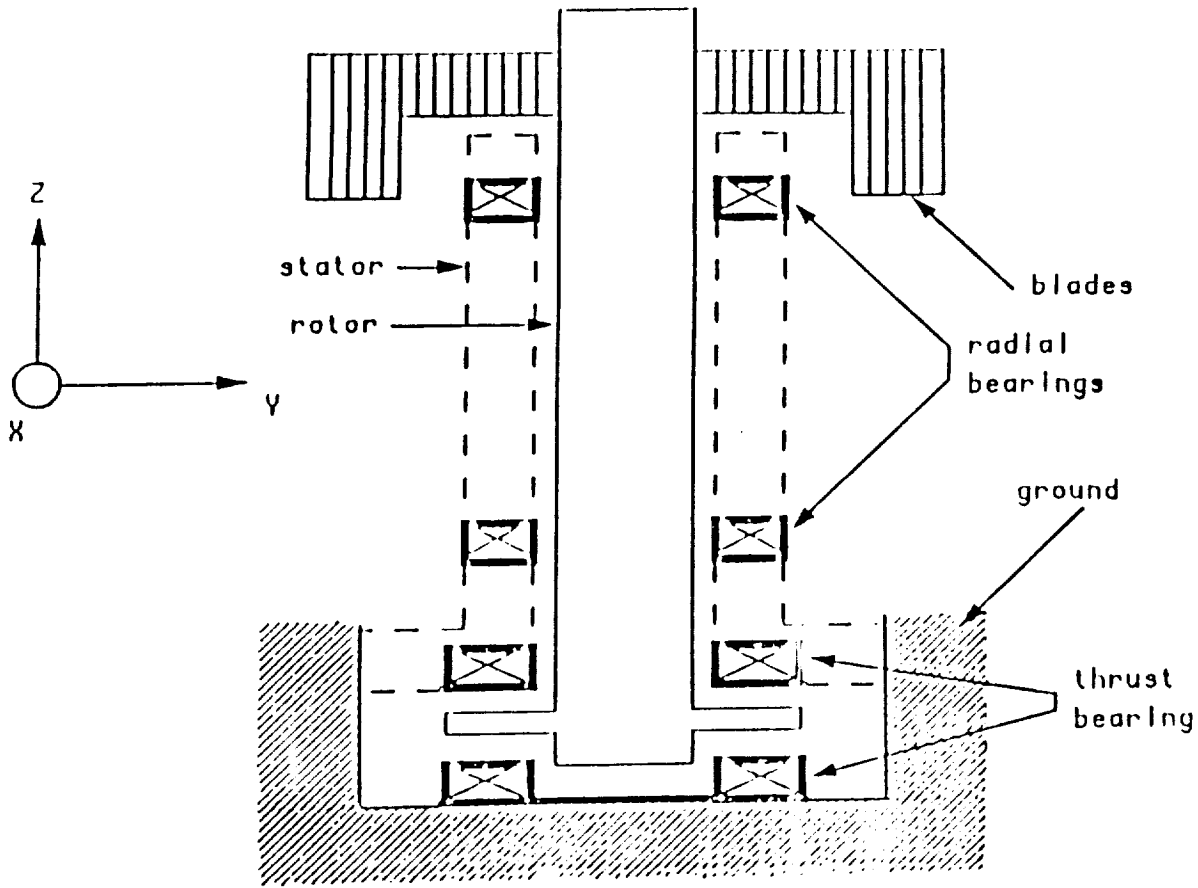Figure 2: A five-axis magnetic bearing

Rotor Mass	=	2.2 Kg
Air gap for thrust bearing	=	400 μm
Air gap for radial bearing	=	250 μm
Maximum current to bearings	=	10 Amp.
Maximum rotational speed	=	45,000 RPM

Table 1: Relevant system parameters

where $x_q \in \mathcal{R}^5$ and $x_r \in \mathcal{R}^5$ represent displacements and velocities of the rotor with respect to the bearing stators respectively. The current inputs to the electromagnets are represented by the vector $u \in \mathcal{R}^5$. The control objective is to levitate the rotor and maintain stability. Also, the control system must reject disturbances under spinning and nonspinning conditions of the rotor.

This plant is multi-input, multi-output with all five degrees of freedom unstable open loop. Disturbances and coupling include forces due to gravity, magnetic actions, unbalance and gyroscopic effects. All of these effects will show up in the vector function G_r. Note that since the magnetic force is proportional to the current squared and inverseley proportional to the gap distance squared the function $G_r(x, u)$ is a nonlinear function dependent on the state x and the control action u.

The variation of the component of G_r in the Z direction in terms of the gap x and control current u are shown in Figures 3, 4, and 5. Figure 3 shows that for a gap of 0.15 mm, the rotor acceleration corresponds to 30 m/sec^2 and 110 m/sec^2 for control currents of 1 and 2 amps respectively. Also, for the same gap opening and with currents levels of 1 and 2 amps, $\frac{\partial g}{\partial x}$ changes from about 0.22 $\times 10^6$ $\frac{1}{\text{sec}^2}$ to about 0.88 $\times 10^6 \frac{1}{\text{sec}^2}$, and $\frac{\partial g}{\partial u}$ changes from about 32 m/amp-sec^2 to about 112 m/amp-sec^2. It is clear that this particular device experiences drastic dynamic changes. Therefore, such dynamic information would be necessary if a conventional control system is used; otherwise the system performance may be acceptable only for some specific operating conditions.

The control algorithm was implemented on a DSP chip as shown in Figure 6. In this experimental setup, we have the option of controlling the system using either a linear analog controller which resides in the compensation block,

or a time delay controller implemented digitally in the DSP board. The position signal for the Time Delay controller is obtained through the test points TP2 and/or TP3 and this signal is then sent into an analog to digital converter (A/D converter) which is linked to the DSP board. The A/D board has an adjustable built in low pass filter where the position signal can be filtered. The control voltage signal is sent out through the D/A converter which has a low pass filter with adjustable cutoff frequency.

The program was written in C and assembly language. The sampling frequency used was around 5 KHz. The computation time for the control algorithm was about 70μsec. The cutoff frequency of the filter for the position signal was kept at 7.2 KHz since the signal from the sensor is already bandlimited to 1 KHz. The position signals are then obtained by the DSP board and the control actions representing currents are sent out through the D/A converter. The D/A converter has a low pass filter with a cutoff frequency set to 1.5 KHz. Although a sampling rate of 5 KHz was adequate for this system, the sampling frequency could be increased up to 15 KHz. This sampling rate could further be increased by optimizing the program code and hence reducing the computation time for the Time Delay Control law.

In this section, we will use the control procedure described earlier to maintain a desired performance. The model reference for the thrust bearing position was chosen as a second order system with a natural frequency of 200 rad/sec and a damping ratio of .707. The experimental data shown in Figure 7 indicate that the actual position response tracks the reference model response very closely. In this case, the position of the rotor moves from 200μm to 0μm which corresponds to the suspended configuration. The error between the desired and actual position trajectories is shown in the same figure and has about less than 10% maximum error. The control current necessary to produce this response is also shown in that same figure with a maximum current of about 1.75 amps. This is an excellent performance considering that the controller has no detailed information about the system. Figure 8 shows the closed-loop frequency response between the reference position and actual position of the thrust bearing. In his case it is clear that the magnitude and phase characteristics are close to those of the reference model selected. In order to check the disturbance rejection properties of the control system, an additional current is injected through the drive amplifiers (Auxiliary input 2 in Figure 6) to create an intentional disturbance force. The frequency of this input is then varied from 0.1 Hz to 10 KHz (sine sweep). In order to check the disturbance rejection properties, we measured the frequency response from the additional current to the position of the rotor. The disturbance rejection properties of the thrust bearing are shown in Figure 9. This curve represents a compliance curve. The controller rejects disturbances up to the bandwidth which is again around 200 rad/sec. The static stiffness is about 100 MN/m and the minimum stiffness is about 300 KN/m at the frequency of 200 rad/sec.

Figure 10 shows the closed loop frequency response for a radial bearing. Again, this response is between the reference position and actual rotor position. This is very similar to that of the reference model. Figure 11 shows the disturbance rejection of the radial bearing when the rotor is at rest and while it is spinning at 10900 RPM, 20100 RPM, 30400 RPM and 34800 RPM. When the rotor is not spinning, the static stiffness is about 200 MN/m and the minimum stiffness is about 500 KN/m. It is clear that the disturbance rejection properties are almost the same for these different operating conditions. Figure 12 shows the effect of using a lower bandwidth of 100 rad/sec. In this case, the disturbance rejection curve moves up indicating a lower stiffness. These data demonstrate that such a control scheme possesses excellent robustness properties.

5   Conclusion

The time delay controller algorithm uses past observations for adaption in controlling systems with unknown dynamics and unpredictable disturbances. The time delay control law is formulated for a class of nonlinear systems with nonlinear input action. The result of stability analysis performed based on the bounded input-bounded output stability approach are presented and interpreted. The control scheme is implemented on a five-degree-of-freedom magnetic bearing. The control performance, evaluated using step responses and disturbance rejection properties, is shown to be excellent despite the complex nonlinearities in the system.

Acknowledgements

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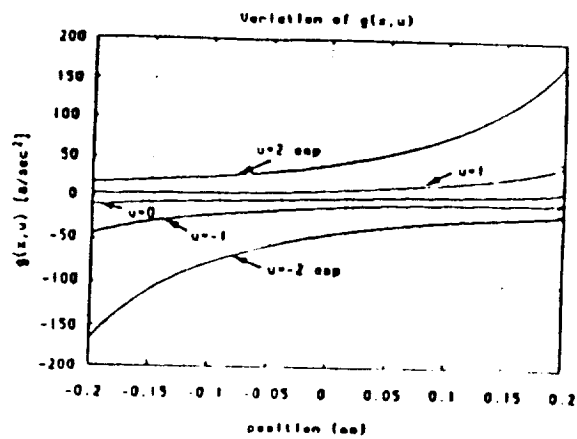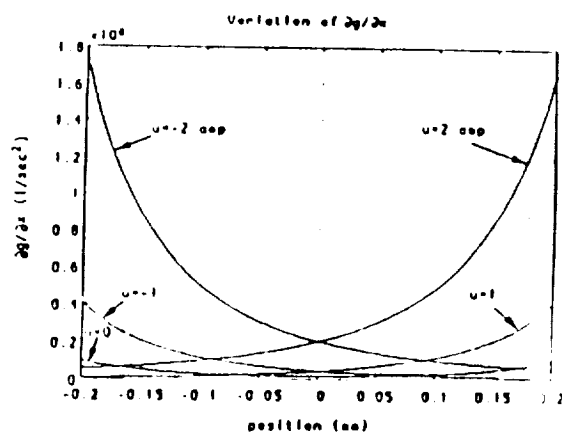
Figure 3: Dynamic changes of the function $g(x, u)$

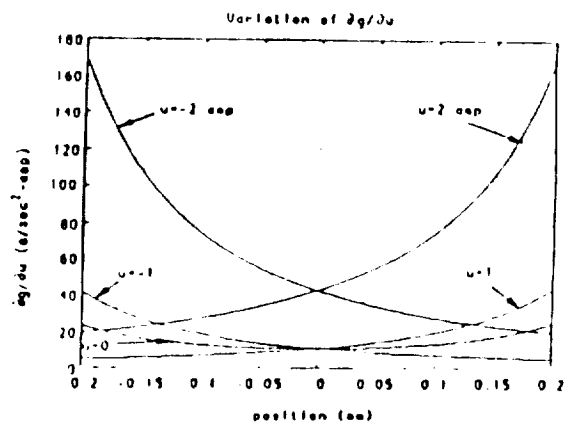Figure 4: Dynamic changes of the function $\frac{\partial g(x, u)}{\partial x}$

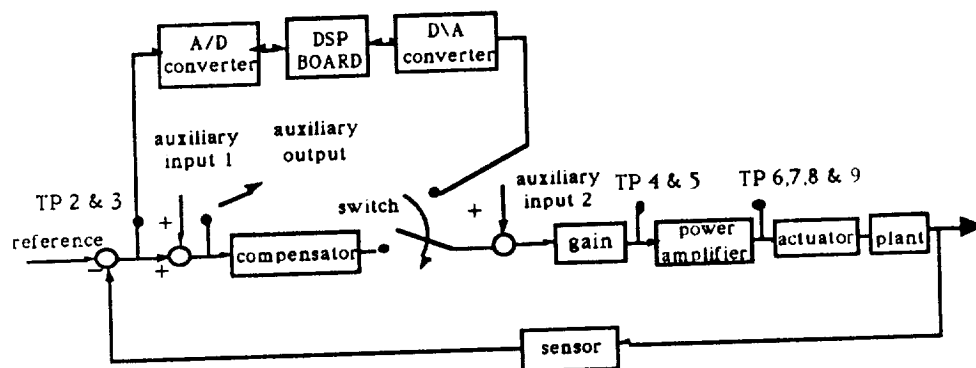Figure 5: Dynamic changes of the function $\frac{\partial g(x, u)}{\partial u}$
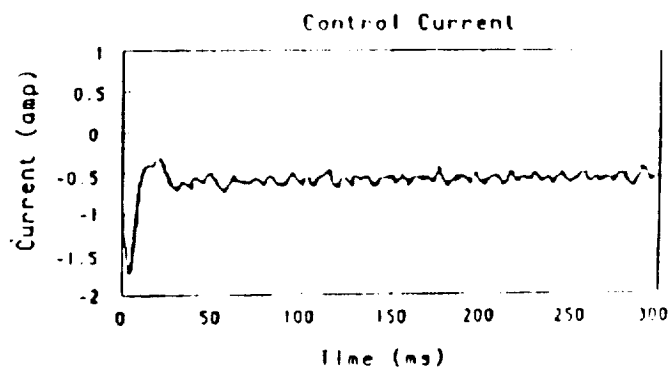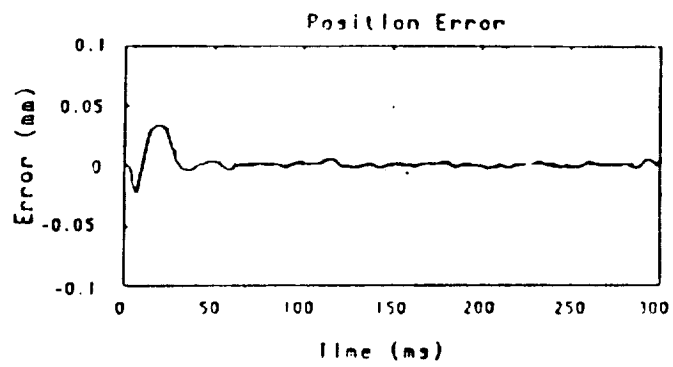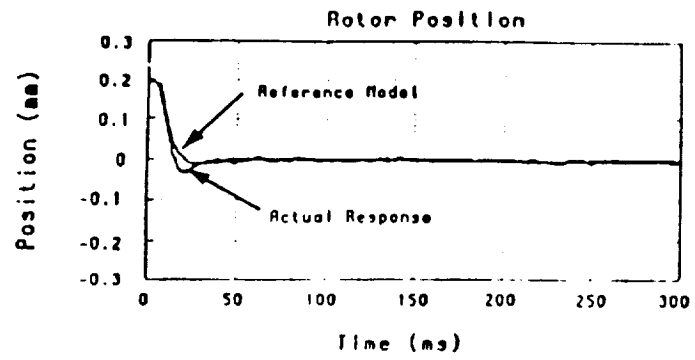
Figure 6: Block diagram representation

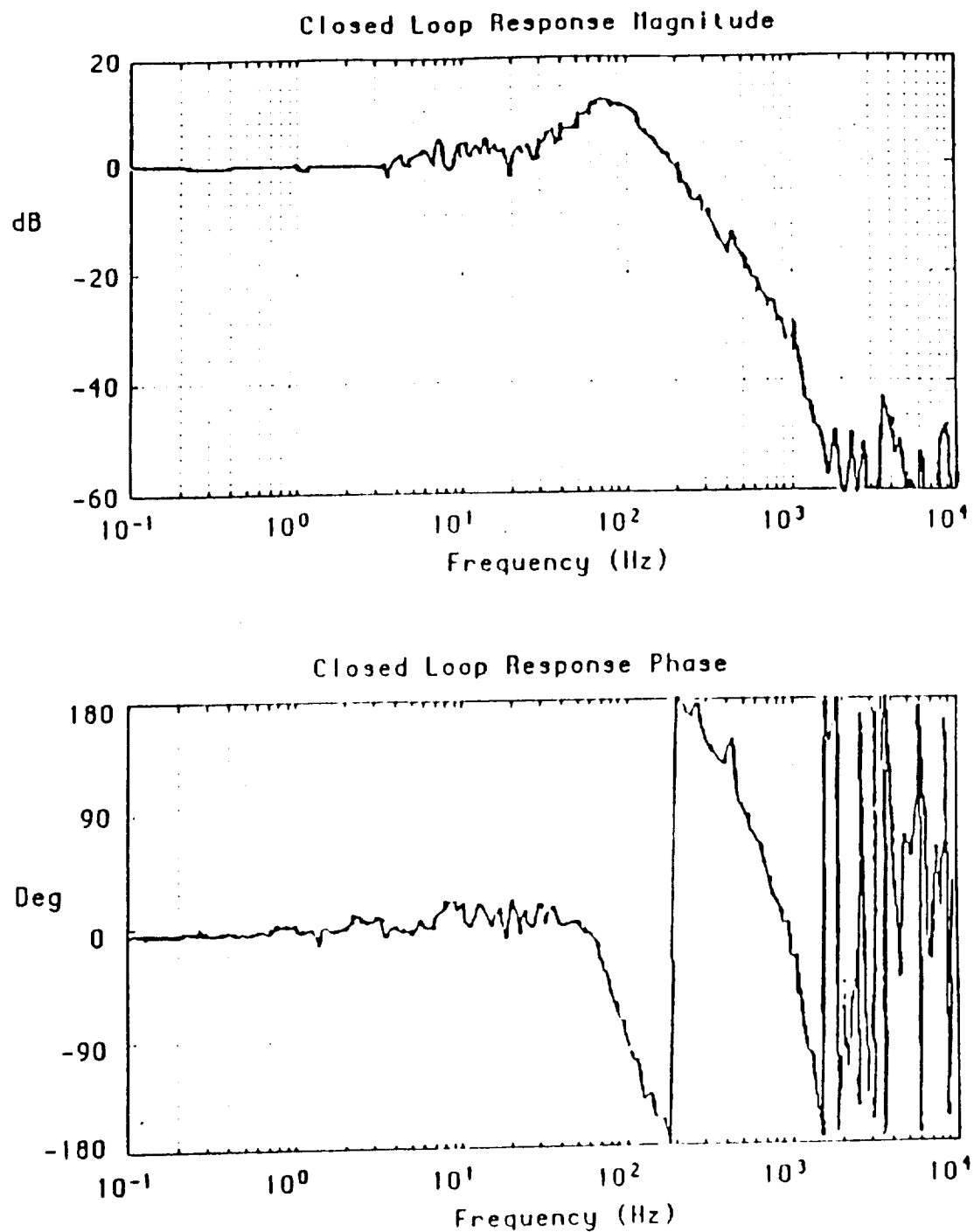Figure 7: Experimental time response data of the thrust bearing

Closed Loop Response Magnitude

Closed Loop Response Phase

Figure 8: Closed loop frequency response of thrust bearing
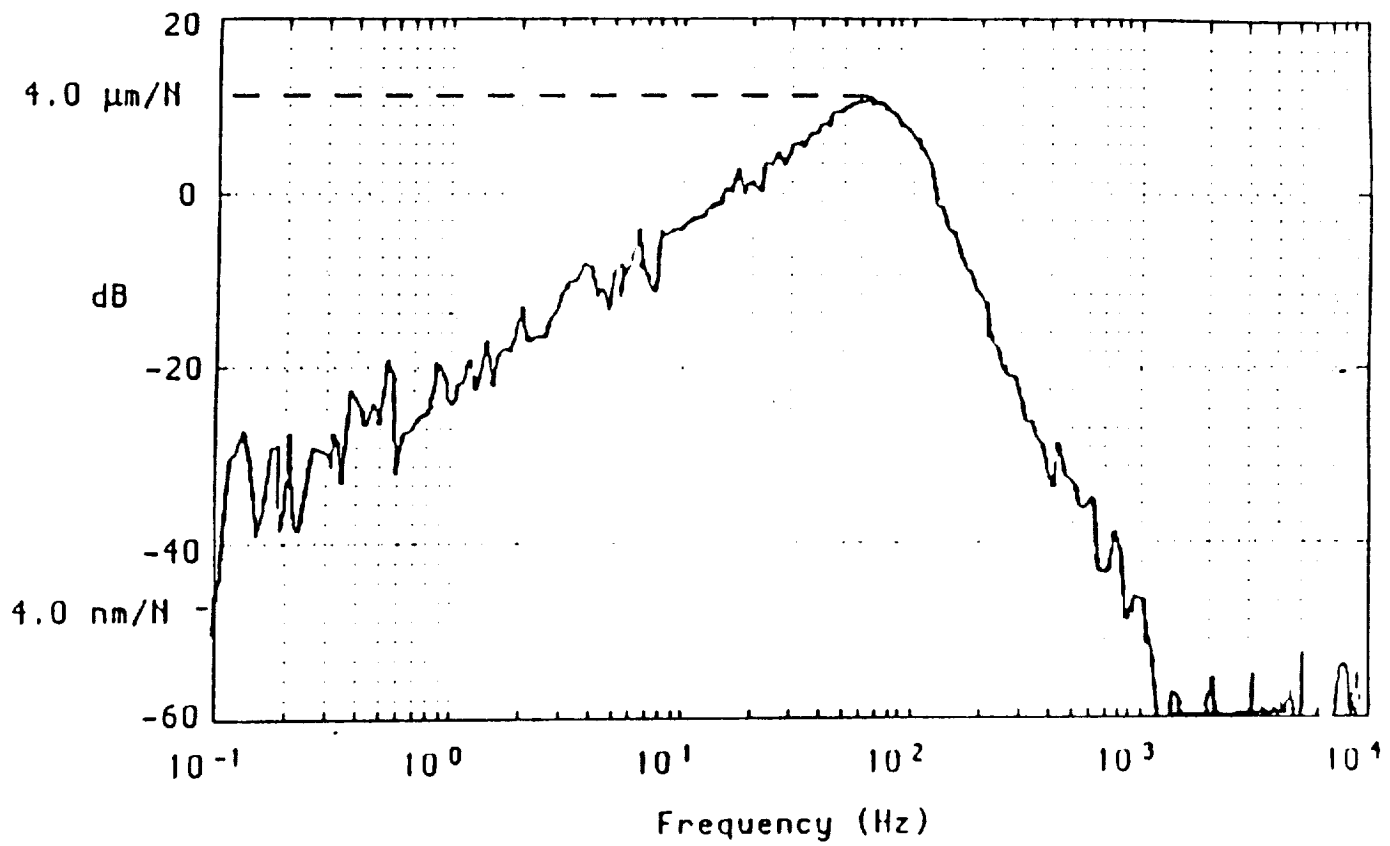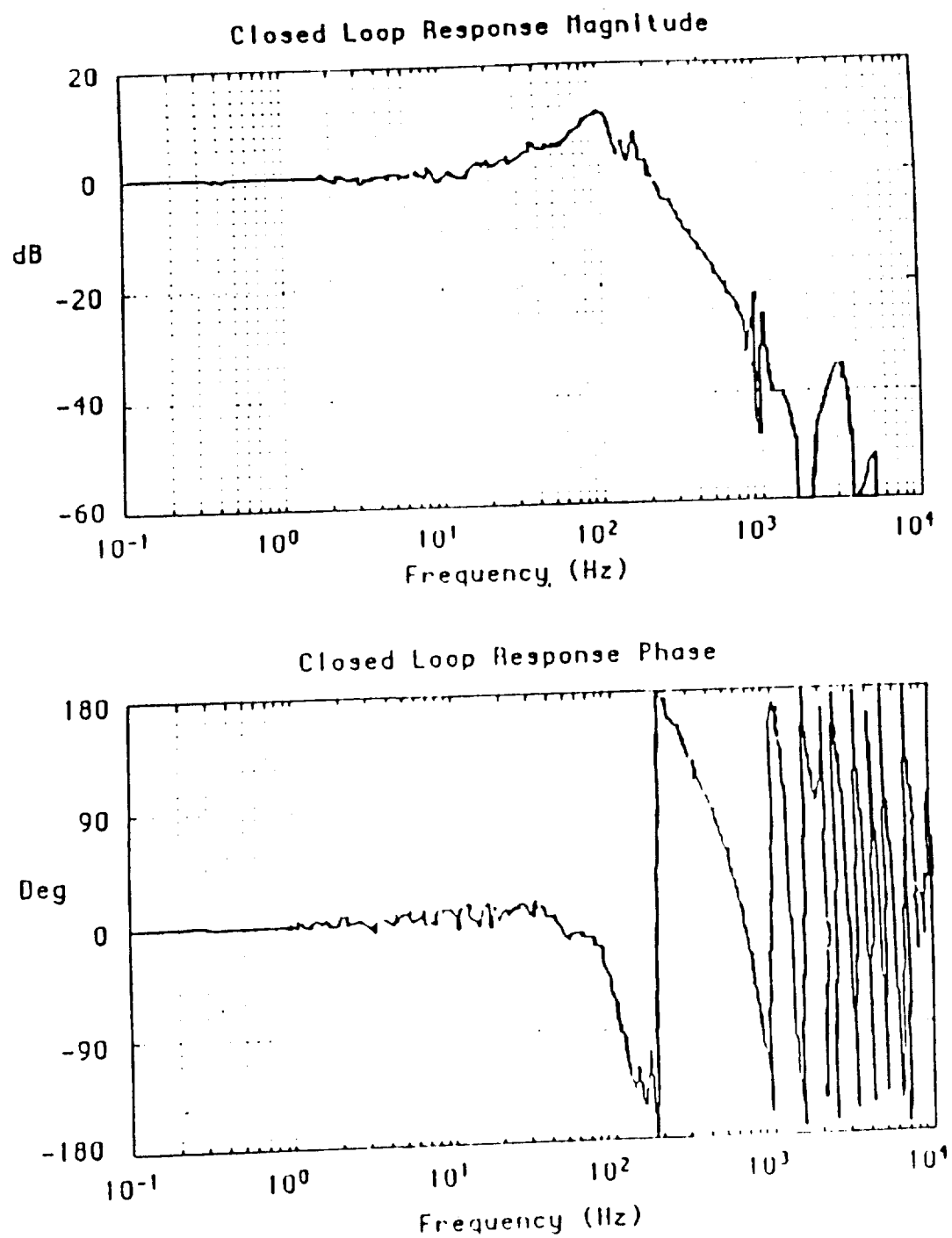
Figure 9: Disturbance rejection of thrust bearing bearing

Closed Loop Response Magnitude

Closed Loop Response Phase

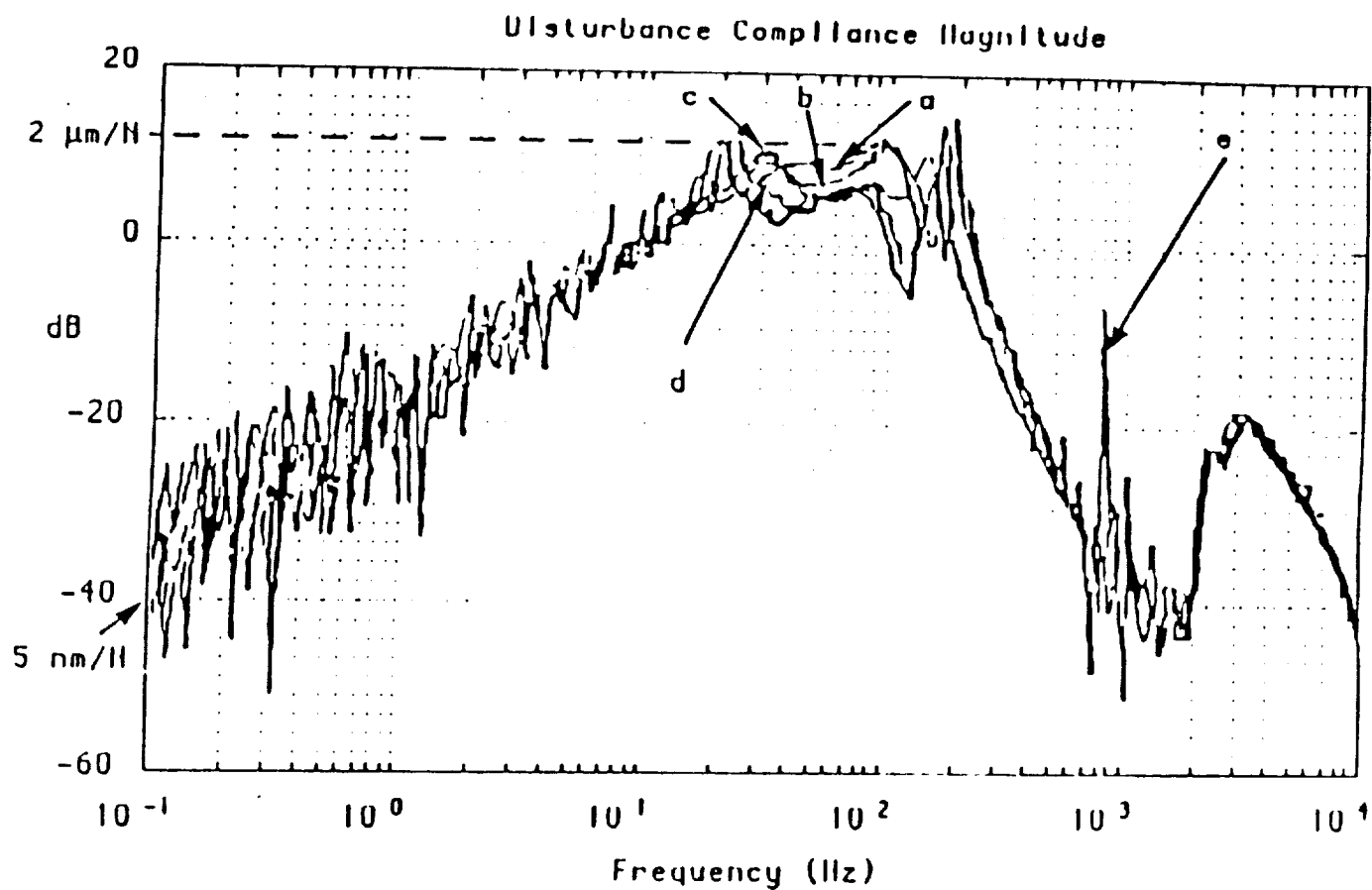Figure 10: Closed loop frequency response of radial bearing

Figure 11: Disturbance rejection of radial bearing (a) at rest, (b) 10,900 rpm, (c) 20,100 rpm, (d) 30,400 rpm, (e) 34,800 rpm.
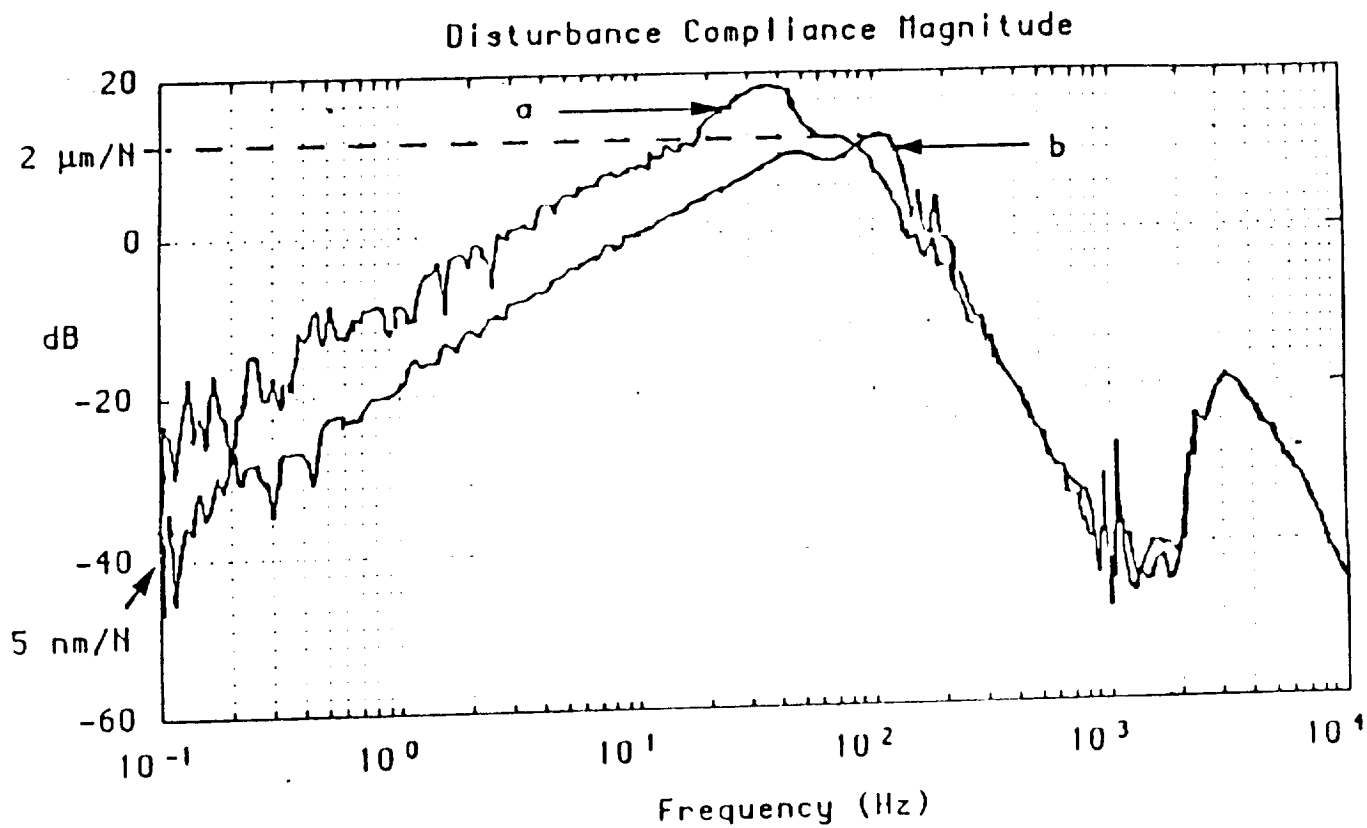
Figure 12: Disturbance rejection of radial bearing with nonspinning rotor (a) bandwidth = 100 rad/sec, (b) bandwidth = 200 rad/sec